

Game Theory and Applications

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A Game Form Model of a Dynamic Multipolar International Conflict among Nations 1–9

Bjorn Ebbesen

Abstract

Inspired by the article Economic constraints, strategic stability and arms reduction by A. Halperin and I. Amit, we derive an n -dimensional array of nonlinear differential equations, reflecting trade among and growth of national economics and use and growth of national military power. Constraints and assumptions are used in an axiomatic manner to derive new functions (economics) or to ensure the applicability of existing military models (Lanchester). Definition and change of payoff functions and solution concepts for this game form allows us to investigate the dynamics of multipolar systems of interacting nations in terms of trade and conflicts. The final application compares two scenarios of international economic cooperation in the form of just distribution of commonly organized growth of economic potential. The scenarios differ in the consideration of military power in negotiations. In the first scenario the threat of possible use of military power is not taken into account, whereas such threatening is considered in the second scenario. A time-consistent, cooperative, non-transferable-utility-based solution concept is derived and used.

An Infiltration Game on a Circumference 11–16

Andrej Yu. Garnaev and Galina Yu. Garnaeva

Abstract

An Infiltrator, starting at a safe base, tries to pass unhindered by the Guard, having a gun, to a safe destination within a time limit. There is a restriction on the number of the Guard's shots. A discrete variant of this zero-sum game on a circle is investigated. The optimal strategies are obtained.

An Approach to Defining Solution Concept in N-Person Nonantagonistic Positional Differential Games 17–26

Anatoly F. Kleimenov

Abstract

One of the basic problems in nonantagonistic positional differential games (NPDGs) theory is the defining of solution concept. Usually, some principle of optimality gives the basis for the solution concept, and the latter is generally based on some additional assumptions. In the paper we try to formalize these additional assumptions so that the choice of concrete solution type can be made on formal basis. Another basic problem in NPDG is choosing a unique representative among multiple solutions of one and the same type. In the paper we introduce two new types of solutions in NPDG: one for the class of closed-loop strategies and the other for the class of feedback strategies. The construction of these solutions essentially employs the marginal control possibilities of all players in NPDG.

The Simple Pursuit by a Few Objects on the Multidimensional Sphere	27–36
<i>Alexander M. Kovshov</i>	

Abstract

This paper makes an attempt to study pursuit games on non-linear manifolds. Discussing this questions we use the terminology given by L. A. Petrosjan (Π -strategy). On the sphere there are two strategies corresponding to the linear Π -strategy. They are Π_1 -strategy and Π_2 -strategy:

Π_1 -strategy prescribes the pursuer to move toward the fixed point named “the center of pursuit” along a geodesic line;

Π_2 -strategy conserves the players’ relative positions an accuracy distance.

This paper shows that Π_2 -strategy is successful in the problem of simple pursuit on the sphere with a general starting position.

Dynamic Games with Optimal Stopping	37–45
<i>Vladimir F. Mazalov</i>	

Abstract

We consider the games $G(x, p, N, K)$, based upon the following form. Imagine two objects in a line (for instance, two yachts) with a distance between them equalling x . For the sake of certainty determine x_1 as the position of object 1 and x_2 - the position of object 2; $x_1 - x_2 = x$. The objects move on a line O_x , with random leaps. Every step players may observe N independent identically distributed random variables from their sequences $y_1^{(1)}, \dots, y_N^{(1)}$ and $y_1^{(2)}, \dots, y_N^{(2)}$ with the continuous distribution function $F(x)$ and density $p(x), x \in R$. The observations continue one after another. Each player may stop them at a random point in time t , being the strategy of a player in this game. The selection of value y_t results in the random leap of an object on the k -step. This procedure may be considered to be a choice based upon the activity of the wind in the yachting illustration. A player wins if his object has advanced farther for K steps than his opponent, $k \geq 1$.

On Two Person Full-Information Best Choice Problem with Imperfect Observation	47–55
<i>Peter Neumann, Zdzislaw Porosinski and Krzysztof Szajowski</i>	

Abstract

The paper deals with the following zero-sum game version of the full-information best choice problem. Two decision makers, Player 1 and Player 2, observe sequentially a known number N of iid random variables from a known continuous distribution with the object of choosing the largest. The random variables cannot be perfectly observed. Each time a random variable is sampled the sampler is informed only whether it is greater than or less than some level specified by him. Each of the players can choose at most one observation. After each sampling players take a decision for acceptance or rejection of the observation. If both want to accept the same observation the priority is given to a specified player, say Player 1. The class of adequate strategies and the suitable gain function for the problem is constructed. In the finite horizon case that the game has solution in pure strategies. A numerical

examples are given. A random number of observations is also investigated. For the geometric N , the equilibrium strategy for Player 1 depends on parameter p of geometric distribution, but it is always pure one. The form of the second Player's strategy depends on parameter p . It is pure for large value of p and is a mixture of two pure strategies.

Stochastic Strategies in Differential Games with Incomplete Information	57–63
<i>Tatyana V. Slobodinskaya</i>	

Abstract

In the paper we consider the simple pursuit zero-sum game with incomplete information and prescribed duration on the plane between the team of pursuers and the team of evaders. In contrast with the pure piecewise constant strategy of the player, we define the stochastic piecewise constant strategy. We formulate the conditions, upon which the epsilon-optimal (SPCS) may be effectively constructed. We also provide two examples of differential zero-sum pursuit-evasion games of prescribed duration with delayed information for both players.

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<i>Igor L. Bratchikov</i>	

Abstract

In the paper we consider the game DIP SONOR, invented at Yakutsk State University in the late 1980s. This is a positional game between two players: a team of evaders and a pursuer. We present a mathematical model of the game as well as the computerization of it that had been undertaken at St.-Petersburg State University. We briefly describe two possible strategies for the pursuer and outline the future extensions of the programming system.

Some Game-Theoretical Models of Conflict in Finance	71–78
<i>Vladimir M. Bure and Oleg A. Malafeyev</i>	

Abstract

Two game-theoretical models of conflict in finance are considered in the paper. The first model gives approach to the solving of a pricing problem. The second model is concerned of a problem of repeated partnerships.

On the Existence of the Coalition Set Core	79–86
<i>Alexander A. Vasin and Vladimir A. Gurvich</i>	

Abstract

The paper considers cooperative games with a given set of admissible coalitions (a coalition set) and superadditive payoff functions. A coalition set is called conciliable if the coalition set core is non-empty for every game of this class. We employ the concept of a balanced set of coalitions (Bondareva, 1963, Shapley, 1964) in order to characterize the conciliable sets. We show that a coalition set is conciliable if and only if every minimal balanced collection included in this set is a partition of the set of players. Similar results are established for set-valued characteristic function and normal form games. The paper discusses several concrete types of conciliable coalition sets.

Evolution and Game-Theoretic Rationality 87–101

Jorgen W. Weibull

Abstract

The usual justifications of non-cooperative solution criteria are rationalistic in nature, building on individual rationality and expectation-coordination postulates. In contrast, the "as if" approach to game-theoretic rationality is evolutionary, arguing that even if strategically interacting agents do not meet these epistemic conditions, their long-run aggregate behavior will nevertheless conform with them, because of the workings of biological, social or cognitive selection processes. The present essay discusses three implications of evolutionary selection dynamics in favour of game-theoretic rationality and four potential obstacles to the general validity of these implications.

A Differential Game Model for Optimal Price Subsidy of New Technologies 103–114

Georges Zaccour

Abstract

We consider a differential game model to investigate the effects of price subsidy over time on the rate of diffusion of a new technology. We assume that the latter is produced by a monopolist whose objective is to maximize his profit stream over the planning horizon $[0, T]$. The government seeks to maximize the total number of installations by time T . The motivation of the government's behaviour is the existence of a net societal benefit in accelerating the diffusion of the new technology. The assumption of a monopolistic market structure stems from patent protection, for instance. The firm controls the price and the government the subsidy rate. It is shown that the timepaths of demand, subsidy, government spending rate and price depend on some product characteristics and cost structure.

Constructions of Nonsmooth Analysis in Numerical Methods for Solving Hamilton-Jacobi Equations 115–131

Svetlana V. Grigorjeva, Alexander A. Tarasjev, Vladimir N. Ushakov and Alexander A. Uspenskii

Abstract

In the paper, finite difference operators (FDO) and approximation schemes (AS) for constructing generalized solutions of Hamilton-Jacobi equations (HJE) with non-convex Hamiltonians are proposed. These constructions are based on results of optimal guaranteed control theory, convex and non-smooth analysis and develop the results on the theory of positional differential games by Krasovski? and Subbotin.

The Distribution of Voting Power in the UN 133–141

William Kerby and Frank Gobel

Abstract

The intention of this paper is to make a contribution to the discussion of the reform proposals based on a power index analysis (PIA). PIA is applied to the present UNO. The reform proposals of a few member states are analysed and a proposal of the authors is introduced.

Multiple Search Game with Nonmovable Hyder 143–151

Teturo Kamae

Abstract

In the paper we describe a generalization of the multiple search game in continuous search time. We consider three zero-sum games: multiple search game without duplication, multiple search game with duplication and multistage search game. We state and prove theorems describing optimal strategies in the three games.

Game Theoretical Model of Harvesting Two Species of Fish .. 153–159

Ekaterina Kunshenko and Viktor Zakharov

Abstract

In this paper we apply method of regularization of the Stackelberg differential game which construct mechanism of payments in the conflict model of harvesting two species by two agents.

Game with Optimal Stopping of Random Walks 161–168

Vladimir V. Mazalov

Abstract

Let us consider a game Γ of two persons, defined on random walks of the following form. Let x_n and y_n are symmetric random walks at the set $E = \{0, 1, \dots, k\}$, beginning in the states a and b from E respectively and absorbing at its ends. Players I and II observe for random walks x_n and y_n and stop them at some moments of time τ and σ . These random variables τ and σ , which we shall suppose to be *stopping times*, are the players' strategies. Then if $x_\tau > y_\sigma$, player I wins; if $x_\tau < y_\sigma$ - player II. But if $x_\tau = y_\sigma$, draw is announced. There is no any information about the opponent's behavior at players' disposal. The situation of equilibrium is sought in such game.

Game with Several Pursuers and One Evader with Discrete Observations	169–184
<i>Arik Melikyan and Odile Pourtallier</i>	

Abstract

The paper presents games with several pursuers and one evader. We consider both cases with the fixed final time, or the game of encounter, and with the non-fixed final time, that is the game of pursuit-evasion. We focus on the situation where the pursuers have observation about the state of the system only at discrete time instants. Two corresponding differential games are formulated with one evader and several pursuers, each pursuer having his own maximal speed and capture radius. A problem is stated to find for a given initial time and position the next observation instant up to which the observation can be omitted without any loss in cost-function. Geometrical algorithm of the problem solution is suggested, which gives for several specific cases an explicit solution and the set of necessary discrete observation moments. The methodology is based on comparison between discrete observations and continuous observation.

Multistage Games with Vector Payoffs	185–191
<i>Leon A. Petrosjan and Tamaki Tanaka</i>	

Abstract

A multistage zero sum two person game with vector payoff is considered. The problem of time consistency and strongly time consistency of optimality principle in this game is investigated. It is proved that the set of all saddle points in any multistage game is a time consistent optimality principle. A counterexample for strongly time consistency is considered. A saddle point, which is not a strongly time consistent, is constructed in this example. The paper also deals with the repeated games with vector payoffs. It is shown that the set of all saddle points in any repeated game is a strongly time consistent optimality principle.

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<i>Sergey E. Miheev</i>	

Abstract

A game of pursuit on the plane with phase constraints is considered. The dynamics of the pursuer and the evader is described by a system of simple motion equations. The pursuer constructs his control on the base of information about his own state, and the state and velocity of the evader. The evader knows only the positions of the pursuer. A pursuit strategy based on attainability domains of the evader is proposed. Properties of connectedness type are introduced for the set of phase constraints. These properties provide construction of the attainability domains for motions governed by the proposed pursuit strategy and guarantee a successful pursuit.

**A Game Theoretic Model of the Decision-Making Process within International Economic Organizations
209–219**

Bernard Gauthier and Leon A. Petrosjan

Abstract

This study proposes a game-theoretical model for analysing decision-making within International Economic Organizations (IEO). The model is based upon the relationship linking member countries, the IEO president, the international bureaucracy and recipient countries within the World Bank and the International Monetary Fund policy-making processes. The optimal conditions selected by the players given the hierarchical structure of delegation are presented. The outcome of the game is a Nash equilibrium under which no actor has an interest in deviating from the solution. The approach is unique in that the solution is found through equilibrium in an auxiliary subgame between the member countries. Previous model presented Romboid structure in the framework of Petrosjan and Danilov (1986). However, this model innovates in the number of the hierarchic levels, and number of the players.